Stress Fields Around Dislocation Arrays With Distributed Cores

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Introduction

Using the basic sum formulae in Section 19-5 from [1] and the nonsingular stress fields from Section 5 of [2], we can derive the stress fields around dislocation arrays composed of dislocation lines with distributed cores.

In all of the formulae in these notes, D is the spacing between dislocations and a is effective radius over which the dislocation core is distributed. All spatial coordinates are in the same orientation as they are in section 19-5 of [1].

Edge Dislocation Array

Burgers Vector Normal to Dislocation Array

The following formulae are written in terms of the reduced variables $X = x/D$, $Y = y/D$, and $A = a/D$.

$$\sigma_{xx}^{array} = -\sigma_0 \sin(2\pi Y) \left[\cosh\left(2\pi\sqrt{X^2+A^2}\right) - \cos(2\pi Y) + 2\pi\sqrt{X^2+A^2}\sinh\left(2\pi\sqrt{X^2+A^2}\right)\right] \quad (1)$$

$$\sigma_{yy}^{array} = -\sigma_0 \sin(2\pi Y) \left[\cosh\left(2\pi\sqrt{X^2+A^2}\right) - \cos(2\pi Y) - 2\pi\sqrt{X^2+A^2}\sinh\left(2\pi\sqrt{X^2+A^2}\right)\right] - \sigma_0 \frac{2A^2\pi}{\sqrt{X^2+A^2}} \sinh\left(2\pi\sqrt{X^2+A^2}\right)\sin(2\pi Y) \quad (2)$$

$$\sigma_{xy}^{array} = \sigma_0 2\pi X \left[\cosh\left(2\pi\sqrt{X^2+A^2}\right)\cos(2\pi Y) - 1\right], \quad (3)$$

where

$$\sigma_0 = \frac{\mu b}{2D(1-\nu)\left[\cosh\left(2\pi\sqrt{X^2+A^2}\right) - \cos(2\pi Y)\right]^2} \quad (4)$$

Burgers Vector Parallel to Dislocation Array

The following formulae are written in terms of the reduced variables $X = x/D$, $Y = y/D$, and $A = a/D$.

$$\sigma_{xx}^{array} = -\sigma_0 2\pi X \left[\cosh\left(2\pi\sqrt{X^2+A^2}\right)\cos(2\pi Y) - 1\right] \quad (5)$$

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$$
\begin{aligned}
\sigma_{yy}^{array} &= -\sigma_0 \frac{X}{\sqrt{X^2+A^2}} \sinh\left(2\pi\sqrt{X^2+A^2}\right)\left[\cosh\left(2\pi\sqrt{X^2+A^2}\right)-\cos(2\pi Y)\right]\left(2+\frac{A^2}{X^2+A^2}\right) \\
&+ \sigma_0 2\pi X\left[\cosh\left(2\pi\sqrt{X^2+A^2}\right)\cos(2\pi Y)-1\right]\left(1-\frac{A^2}{X^2+A^2}\right) \qquad (6)\\
\sigma_{xy}^{array} &= \sigma_0 \sin(2\pi Y)\left[\cosh\left(2\pi\sqrt{X^2+A^2}\right)-\cos(2\pi Y)-2\pi\sqrt{X^2+A^2}\sinh\left(2\pi\sqrt{X^2+A^2}\right)\right] \\
&+ \sigma_0 \frac{2A^2\pi}{\sqrt{X^2+A^2}} \sinh\left(2\pi\sqrt{X^2+A^2}\right)\sin(2\pi Y), \qquad (7)
\end{aligned}
$$

where

$$
\sigma_0 = \frac{\mu b}{2D(1-\nu)\left[\cosh\left(2\pi\sqrt{X^2+A^2}\right)-\cos(2\pi Y)\right]^2} \qquad (8)
$$

Screw Dislocation Array

The following formulae are written in terms of the reduced variables $X = x/D$, $Y = y/D$, and $A = a/D$.

$$
\begin{aligned}
\sigma_{xz}^{array} &= -\sigma_0 \sin(2\pi Y)\left[\cosh\left(2\pi\sqrt{X^2+A^2}\right)-\cos(2\pi Y)\right] \\
&- \sigma_0\left(\frac{A^2\pi}{\sqrt{X^2+A^2}}\right)\sinh\left(2\pi\sqrt{X^2+A^2}\right)\sin(2\pi Y) \qquad (9)\\
\sigma_{yz}^{array} &= \sigma_0\left(\frac{X}{\sqrt{X^2+A^2}}\right)\sinh\left(2\pi\sqrt{X^2+A^2}\right)\left[\cosh\left(2\pi\sqrt{X^2+A^2}\right)-\cos(2\pi Y)\right]\left(1+\frac{A^2}{2\left(X^2+A^2\right)}\right) \\
&+ \sigma_0 2\pi X\left(\frac{A^2}{2\left(X^2+A^2\right)}\right)\left[\cosh\left(2\pi\sqrt{X^2+A^2}\right)\cos(2\pi Y)-1\right], \qquad (10)
\end{aligned}
$$

where

$$
\sigma_0 = \frac{\mu b}{2D\left[\cosh\left(2\pi\sqrt{X^2+A^2}\right)-\cos(2\pi Y)\right]^2} \qquad (11)
$$

A Stress Fields for Edge Dislocation with Burgers Vector (0,1,0)

The following stress fields are computed by applying a coordinate transformation that maps $\hat{\mathbf{x}}$ to $-\hat{\mathbf{y}}$ and $\hat{\mathbf{y}}$ to $\hat{\mathbf{x}}$.

$$
\begin{aligned}
\sigma_{xx}^{ns} &= \frac{\mu b}{2\pi(1-\nu)}\frac{x}{\rho_a^2}\left[1-\frac{2\left(x^2+a^2\right)}{\rho_a^2}\right] \qquad (12)\\
\sigma_{yy}^{ns} &= -\frac{\mu b}{2\pi(1-\nu)}\frac{x}{\rho_a^2}\left[1+\frac{2\left(y^2+a^2\right)}{\rho_a^2}\right] \qquad (13)\\
\sigma_{xy}^{ns} &= \frac{\mu b}{2\pi(1-\nu)}\frac{y}{\rho_a^2}\left[1-\frac{2x^2}{\rho_a^2}\right] \qquad (14)
\end{aligned}
$$

References

[1] J. P. Hirth and J. Lothe. *Theory of Dislocations*. Krieger Publishing Company, Malabar, FL, 1982.

[2] W. Cai, A. Arsenlis, C. R. Weinberger, and V. V. Bulatov. A non-singular continuum theory of dislocations. *Journal of the Mechanics and Physics of Solids*, 54:561–587, 2006.